**Example: Distance through which the Earth rotates around Sun**

Let’s the distance the Earth rotates about the Sun.

The Earth travels in a circle, approximately, about the Sun. After one complete revolution, then, it has rotated through an angle 2π. Therefore it has travelled through a distance,



where r is the distance between the Earth and the Sun. The distance between the Earth and Sun can be found on Wikipedia. But we can estimate it using the following information. It takes about 8min. for the light coming from the Sun to hit the Earth. Therefore Earth is d = vΔt = (3×108)(8×60) = 1.44×1011m away. (3×108m/s is the speed of light). So then it travels through a distance of:



**Example: Speed of the Earth around the Sun**

Let’s calculate the speed of the Earth around the Sun. First, we know the angular velocity of the Earth around the Sun. This is:



The distance between the Earth and Sun can be found on Wikipedia. But we can estimate it using the following information. It takes about 8min. for the light coming from the Sun to hit the Earth. Therefore Earth is d = vΔt = (3×108)(8×60) = 1.44×1011m away. (3×108m/s is the speed of light). So then,



**Example: clock**

How fast is the tip of the seconds hand moving if it is 15cm long? How fast is the middle of the seconds hand moving?

The angular velocity of the seconds hand is:



The tip of the seconds hand would have a speed of:



The middle of the seconds hand would have a speed of:



**Question 6**. A pilot undergoing flight training is accelerated around a circle of radius r = 10m, at a rate of as = 0.75m/s2. What will be the magnitude of his acceleration after he has completed a quarter of a revolution? Note g = 9.8m/s2.

He’ll complete one revolution when



At which point his velocity will be:



His acceleration at this time will be:



**Example**

Suppose you constantly accelerate a merry-go-round, starting from rest, through 10 revolutions in 25s. What is its final angular velocity?



**Example: clothes dryer**

Suppose a clothes dryer when turned on builds up to an angular velocity of 2 rotations per second in 5s. What is its angular acceleration? If a piece of clothing is at the edge of the dryer during this time what is its tangential acceleration? Suppose the radius of the dryer is 60cm.



The tangential acceleration would be:



**Example: Relating vt to ω**

Suppose your car is traveling at 70mph. How quickly are your tires rotating, assuming that they have a diameter of 80cm? What is the frequency of rotation?

If you’re traveling at 70mph (70/2.25 = 31m/s), then the surface of the tires is also traveling at this rate. So vt = 31m/s. Now since r = 0.40m, the angular velocity of the tires is:



So the angular velocity of the tires is 77.5 radians/second (the radians are typically not explicitly indicated in angular velocity numbers). Since a revolution corresponds to 2π rad, the frequency of rotation is:



so it rotates 12.3 times per second.

**Example: Relating at to α**

Suppose your car accelerates from 0 to 60mph (27m/s) in 7s. If your tires have a diameter of 80cm, through how many revolutions do they turn during this time?

One way to answer is the following. Determine the distance, Δs, through which the tires revolve, and then convert this to Δθ. So, using , we can determine through what distance the car travels during this time. First determine the acceleration of the car.



then to determine the displacement of the car, use,



Now the distance the car travels is exactly equal to the distance through which the rim of the tires revolves –Δs in other words. So Δs = 94.4m, and to determine the angle through which the tires revolve, we can use,



so to determine the number of revolutions, divide by 2π,



Another way to do this is to determine the initial, final angular velocities, and the angular acceleration of the tires. ω0 = 0 since the car starts from rest. The final angular velocity, ω = vt/r = 27/0.4 = 67.5s-1. The angular acceleration is α = at/r. Now at = a since the tire rims rotate through every piece of distance that the car traveles. So then, at = 3.86m/s2. This corresponds to an angular acceleration of α = at/r = 3.86/0.4 = 9.65s-2. Therefore:



so we get the same answer.

**Problem 5**

(a) A fan is rotating counterclockwise at a rate of 2 rev/s. When you turn off the power it decelerates at a rate of 0.15 rev/s2. Write an expression for its angular velocity ω and angular position θ as a function of time (you can assume the initial angle is θ0 = 0, and convert units to radians).

So first ω0 = 2rev/s = 2(2πrad)/s = 4π rad/s = 12.6 rad/s. And α = −0.15 rev/s2 = −0.15∙2π rad/s2 = −0.94 rad/s2. So filling these in we have:



(b) When will the fan stop?

The fan will stop when,



(c) If the length of the fan blade is 0.6m, what will be the magnitude of the acceleration of the tip of the fan blade at t = 10s?

In order to figure out a, we will need as and ac. as is given by as = rα = (0.6)(0.94) = 0.56 m/s2. ac is given by ac = vs2/r = (rω)2/r = rω2. We will need ω therefore at time t = 12s. This is:

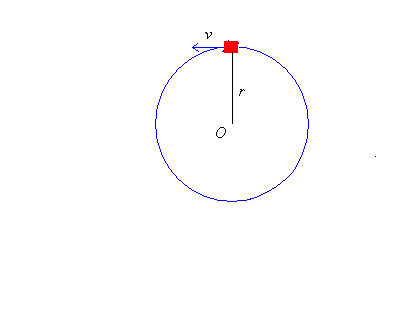


and so ac = (0.6)(1.32)2 = 1.05 m/s2. Combining the two we get:



**Example**

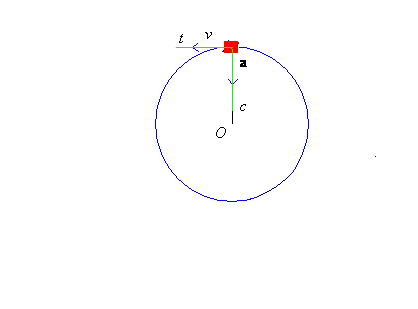
Consider a car driving along a circular track with radius r = 5m, at a constant speed of v = 10m/s. What is its acceleration (give magnitude and direction) when it is at the top of the circle?



The acceleration would be:

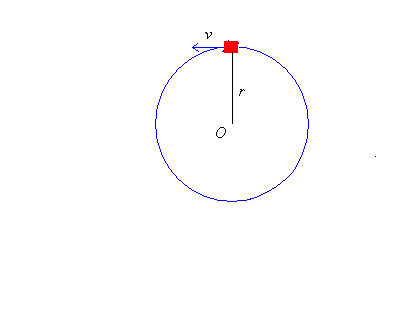


Note that since the speed is not changing, there is no acceleration in the tangential direction (dv/dt = 0). So the acceleration is completely in the centripetal direction (i.e., towards the center of the circle), illustrated below,



**Example**

Consider the car again, rotating in a circle with radius r = 5m. Suppose that it starts at the top of the circle, at rest, but then increases its speed as it goes along at a rate of 2m/s2. What is its acceleration just as it makes one complete revolution?



The acceleration would be:



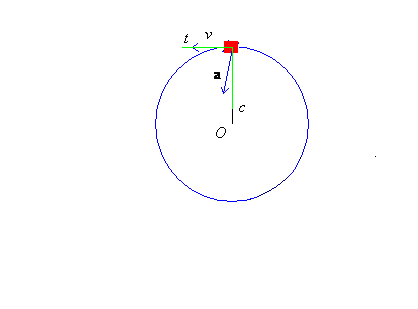
The 2 comes from the fact that its speed is increasing, so there is a tangential component to the acceleration. The centripetal component is unknown at the moment since we don’t know what its speed is. But we can determine this. We know the initial speed v0 = 0, and we want the final speed v, given a = 2m/s2 and that it has travelled a distance of Δx = 2πr (the circumference of the circle). So we use the equation,



Filling this in, we have,



Note that since the speed is not changing, there is no acceleration in the tangential direction (dvt/dt = 0). The acceleration is illustrated below,



The magnitude of the acceleration is:



and the direction is:



**Question 6**. A pilot undergoing flight training is accelerated around a circle of radius r = 5m, at a rate of as = 0.75m/s2. When will his centripetal acceleration be equal to 5 times the acceleration due to gravity? Note g = 9.8m/s2.

Centripetal acceleration is given by ac = vs2/r. And vs = v0s + ast → vs = 0 + (0.75m/s2)t. Plugging this into the ac formula and equating to 5g, we get:



1. Suppose you’re an F-16 fighter pilot flying at v = 600m/s. If you can execute the turn with a maximum centripetal acceleration of ac = 9g, where g = 9.8m/s2, what would be your minimum turning radius?

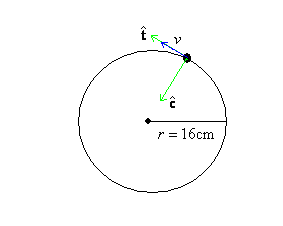
The acceleration will be given by:



**Problem 4.6**

What is the magnitude of the acceleration of a speck of clay on the edge of a potter’s wheel turning at 45 rpm (revolutions per minute) if the wheel’s diameter is 32cm?

Let’s draw the speck rotating in its circle,



We draw in the tangential and centripetal axes. Remember **t** points tangent to the direction of the velocity (which we’ve arbitrarily made to go counter clockwise), and **c** points towards the center of the circle.its rotating about. Now the acceleration is:



The velocity we can obtain from the fact that it makes 45 revolutions in 1 minute. So using v = d/t,



and then,



The tangential acceleration is the change in *speed* over time. But this is 0 since it goes at a constant speed around the circle.



The total accceleration would be:



1. A rotating merry-go-round accelerates uniformly from rest to make one complete revolution in 2s. If a person sitting on it is 1.5m from the center, what is the magnitude of her total acceleration at t = 3s?

Her angular acceleration is:



and her angular velocity at this time is:



So her total acceleration is:



2. If a car tire has a radius of 0.5m, an initial angular velocity ω­0 = 0, and angular acceleration of α(t) = 8t3 rad/s2, how far has it traveled after 20s?

The angular velocity as a function of time is:



and so the angular position as a function of time is:



and the arc length (distance travelled as a function of time is)



So the distance traveled after 20s is:



6. If a car tire has a radius of 0.4m, and an angular acceleration of α(t) = (1+t) rad/s2, how far has it traveled after 20?

Integrate once to get ω(t)



And now the angle as a function of time is:



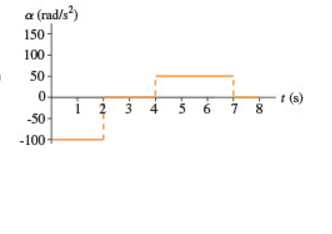
And so the distance traveled is:



and so the distance traveled after t = 12s is:



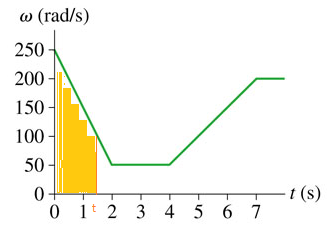
**Question 7**. A blender’s angular acceleration is given by the following graph. Assuming it starts from rest (a) when is the blender speeding up? and when is it slowing down? (b) what maximum angular speed does the blender attain during the 8s time interval? Which direction is it turning in? You may assume that it starts from rest.



Part a) its speeding up between 0 and 2s, slowing down between 4 and 7s.

Part b) angular velocity is equal to area under the curve. As angular speed is equal to the absolute value of the angular velocity, it’s maximum angular speed will be attained when the area under the curve has attained its maximum absolute value. This will occur after 2s, when the angular velocity is (-100rad/s2)(2s) = -200rad/s, which corresponds to an angular speed of 200rad/s. In contrast, the angular velocity after 7s will be (-100rad/s2)(2s) + (50rad/s2)(3s) = -50 rad/s, which corresponds to an angular speed of 50 rad/s.

**Question 7**. The angular velocity of a flywheel is illustrated below. (a) What was the wheel’s initial angular acceleration? (b) When did the wheel complete 30 revolutions?



(a) initial angular acceleration of the wheel is the initial slope of the graph. And this is approximately: α = dω/dt = (50rad/s – 250rad/s)/(2s – 0s) = -100rad/s2.

(b) The wheel will complete 30 revolutions when θ = 30(2π) = 60π rad. This will happen at some time t, illustrated above. According to formula θ = θ0 + ∫ωdt = 0 + ∫ωdt, we need the area under the graph from 0 to t to equal 60π. Area under graph is area of the highlighted trapezoid. And this is:



**Question 8.** To throw the discus, the thrower holds it with a fully outstretched arm. Starting from rest, he begins to turn with a constant angular acceleration, releasing the discus after making one complete revolution. If the radius of this circle is about 0.65m, and it takes about 1.2sto complete the revolution, (a) what is the throwers final angular velocity in rad/s? (b) what is the final tangential velocity of the discus?

(a) To get final angular velocity, we plug what we know into the two constant acceleration angular formulas. These are: θ = θ0 + ω0t + (1/2)αt2 → 2π = (1/2)α(1.2)2 → α = (2π)(2)/(1.2)2 = 8.7rad/s2. And so final angular velocity will be ω = ω0 + αt = 0 + (8.7)(1.2) = 10.44 rad/s.

(b) speed of the discus will follow from vs = rω = (0.65m)(10.44 rad/s) = 6.8m/s.

**Question 2.** To throw the discus, the thrower holds it with a fully outstretched arm. Starting from rest, he begins to turn with a constant angular acceleration, releasing the discus after making one complete revolution. If the radius of this circle is about 0.75m, and it takes about 1.5sto complete the revolution, what is the final tangential velocity of the discus?

To get final *angular* velocity, we plug what we know into the two constant acceleration angular formulas. These are: θ = θ0 + ω0t + (1/2)αt2 → 2π = (1/2)α(1.5)2 → α = (2π)(2)/(1.2)2 = 5.6 rad/s2. And so final angular velocity will be ω = ω0 + αt = 0 + (5.6)(1.5) = 8.4 rad/s. The speed of the discus will follow from vs = rω = (0.75m)(8.4 rad/s) = 6.3m/s.

6a. Two runners, Martha and George are racing around a circular track, whose radius is 60m. Suppose Martha runs counterclockwise, starting from rest with an acceleration of 0.2m/s2, and George runs clockwise at a constant speed of 6m/s. Write respectively an expression for Martha and George’s arc velocity vs, and arc position s, as a function of time.

For Martha,



For George,



6b. When will they meet the first time? When will they meet the second time?

They meet the first time when …



They meet the second time when,



6c. How far around the track will the George have gone when they meet the first time?

His arc position will be:



So he will have run 230m around the track (clockwise)

6d. When will Martha’s centripetal acceleration be equal to 0.4m/s2? What will be the magnitude of her total acceleration at this time?

Her centripetal acceleration will be ac = 0.4 m/s2 when,



and her arc velocity will equal 4.9 when (recalling her vs = 0.2t),



The magnitude of her total acceleration will be,



7a. A fan is rotating counterclockwise at a rate of 2 rev/s. When you turn off the power it decelerates at a rate of 0.15 rev/s. Write an expression for its angular velocity ω and angular position θ as a function of time (you can assume the initial angle is θ0 = 0).

We have:



7b. When will the fan have stopped? Through how many revolutions will it have rotated through during this time?

The fan will stop when



and at this time, its angular position (in rev) will be:



7c. If the length of the fan blade is 0.6m, how fast is the tip moving after 3.7s?

For this we will use the formula vs = rω. We know r = 0.6m. So now we need ω, for which we will use the formula ω = 2 – 0.15t = 2 – 0.15(3.7) = 1.45. But observe that this is 1.45 rev/s, and we must convert this to rad/s to use the formula vs = rω. So we say ω = 1.45 rev/s = 1.45 rev/s ∙ 2π rad/rev = 9.1rad/s. Now filling this into vs we get vs = (0.6)(9.1) = 5.46 m/s.

**Question 8**. A computer hard disk starts from rest, then speeds up to 7000 rpm in 3s. How many rotations has it gone through by this time?

First we ought to calculate the angular acceleration. This is given from ω = ω0 + αt 🡪 7000 rpm = 0 + α(3s) 🡪 α = 7000rpm/3s = 2333 rpm/s = 2333 rev/60s/s = 38.9 rev/s2. Now we use the angle equation: θ = θ0 + ω0t + (1/2)αt2 🡪 θ = 0 + 0(3) + (1/2)(38.9)(32) = 175 rev.

1. When you turn on your fan, it goes from an angular speed of 0 rev/s, to an angular speed of 5 rev/s in a time span of 4s. What is the angular acceleration of the fan in rev/s2, or in rad/s2?



2. What was the fan’s angular velocity at t = 3s?



3. What is the angular displacement of the fan in these first 4s (in rev, or in rad)?



4. If the fan blade has a length of 45cm, how fast is the tip of the blade moving after 4s?

To get the speed of the blade we will use the formula v = rω. But first we must convert ω to rad/s. So:



So the speed of the tip of the blade is:



(remember that a ‘rad’ doesn’t have any dimensions and so it can just go away, meaning m ∙ rad /s

= m / s).

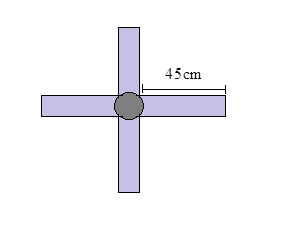
5. What *distance* has the tip of the fan blade rotated through after 4s?

The distance is given by the arc length s = rθ. But θ must be converted to rad to use this formula so we have:



6. Treat the fan as a set of 4 rectangles, each of length 45cm, and mass 0.83kg. What is the moment of inertia of the fan about its central axis?

The fan looks like this:



Remember the formula for the moment of inertia of a rectangle rotating about its end point was I = (1/3)mL2. So each blade has a moment of inertia I = (1/3)(0.83 kg)(0.45 m)2 = 0.056 kg∙m2. Since there are 4 blades, the total I is 4(0.056 kg∙m2) = 0.224 kg∙m2.

7. What torque must the fan motor provide, to accelerate the fan with the angular acceleration you calculated above?

The torque is given by N2L for rotation:



**Question 7.** A fan is rotating counterclockwise at a rate of 2 rev/s. When you turn off the power it takes 10s for it to come to rest. (a) How revolutions has it turned through during this time? (b) For a point on the tip of the fan blade, when is the magnitude of its centripetal acceleration equal to the magnitude of its tangential acceleration?

(a) We have



and



It comes to rest when ω = 0 → 2 + α(10) = 0 → α = -1/5 rev/s2. Pluggint this into θ(t) expression and solving for t gives us:



1. A Ferris wheel, starting from rest, accelerates up to its operating angular velocity of ω = 5 rev/minute in one rotation. So what was its angular acceleration (in rad/s)? A

First let’s write the final angular velocity in terms of rad/s.



To get α, we can use the equation



2. In the problem above, how long did it take for the Ferris wheel to make the first revolution (in seconds)? C

We can use the equation,



3. A car is driving along with a speed of 30m/s, when it suddenly brakes and comes to rest in 3s. If the radius of the tires on the car is 50cm, then how many revolutions do the tires make in those 3s? D

We’ll first figure out how far the car has gone,



and this is also the arc length through which the tires rotate. So then the angle through which they rotate is:



and this corresponds to a number of revolutions



**Problem 5.1 Solution**

Suppose a helicopter blade has a radius of 2m. If it accelerates from rest to a rate of rotation of 1200 revolutions per minute (rpm) in 10s…

(a) what is the angular acceleration of the tip? of the point midway between the center and the tip?

The angular acceleration is:



Note that we have converted rpm’s to rad/s (1 revolution/minute = 2π radians/60seconds). The angular acceleration of the midway point would be the same

(b) what is the tangential acceleration of the tip of the blade? of the point midway between the center and the tip?

The tangential acceleration at the tip is:



and at the midway point, it is:



(c) what is the tangential speed of the tip of the blade after 10s? of the point midway between the center and the tip?

The tangential speed of the tip is:



and at the midway point,



(d) through what angle has the blade rotated in those 10s? of the point midway between the center and the tip?

The angular displacement can be determined from,



Call the initial angle 0. And then the total angle rotated through will be θ, which is:



The midpoint will have rotated through the same angle.

(e) through what arc length has the blade rotated in the 10s? of the point midway between the center and the tip?

The arc length travelled is:



and the midpoint will have travelled,



(f) what is the angular velocity of the blade after 2s? of the point midway between the center and the tip?

We can determine the angular velocity after 2s via:



The angular velocity of the midpoint would be the same.

(f) what is the angular velocity of the blade after 50 revolutions? of the point midway between the center and the tip?

50 revolutions corresponds to Δθ = 50·2π radians = 314 radians. We can determine the angular velocity after this displacement using the equation,



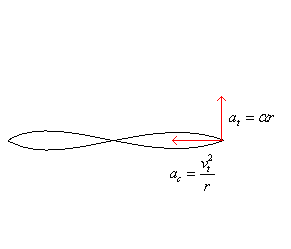
Therefore,



The angular velocity of the midpoint would be the same.

(i) what is the total translational acceleration of the tip of the blade after 50 revolutions? of the point midway between the center and the tip?

The total translational acceleration would be the (vector) sum of the tangential and centripetal accelerations.



The translational acceleration we’ve already determined in part (b) to be at = 25.2 m/s2. To determine the centripetal acceleration, we need to know the tangential velocity after 50 revolutions. In part (i) we determined the angular velocity after 50 revolutions, and so the tangential velocity will be just,



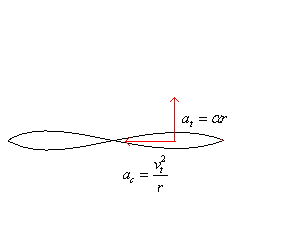
and so,



Therefore the total acceleration is:



As for the middle, we’d have,



The tangential and centripetal accelerations both change since r changes. In part b, we previously determined at to be: at = 12.56 rad/s2. To get the centripetal acceleration we need vt. From vt = ωr, this is:



and so,



and so the total acceleration will be:



since ac is so much larger than at (in these two cases), the total acceleration is approximately just the centripetal acceleration. But this isn’t generally true.

**Problem 5.2 (8.4 in text)**

The blades in a blender rotate at a rate of 6500 rpm. When the motor is turned off during operation, the blades slow to rest in 3.0s. What is the angular acceleration as the blades slow down?

We can calculate the angular acceleration from,



**Problem 5.3 (8.5 in text)**

A child rolls a ball on a level floor 3.5m to another child. If the ball makes 15.0 revolutions, what is its diameter?

Since it rolls through a distance of 3.5m, then means that the arc length through which it revolves is 3.5m. The angle it rotates through is Δθ = 15·2π = 30π radians. So the radius of the ball will be given by,



Therefore the diameter is:



**Problem 5.4 (8.8 in text)**

A rotating merry-go-round makes one complete revolution in 4.0s. (a) What is the linear speed of a child seated 1.2m from the center. (b) What is her acceleration (give components)?

The angular velocity of the child is:



The linear (tangential) speed is:



When they ask for acceleration, they mean, translational acceleration. The total translational acceleration for a point undergoing rotational motion is:



at = 0 in this case, since the speed of the angular acceleration of the merry-go-round is 0 (presumably it is now rotating at a constant rate). As for the centripetal acceleration, we have,



so the total acceleration is:



**Problem 5.5 (8.17 in text)**

Pilots can be tested for the stresses of flying high speed jets in a whirling ‘human centrifuge’, which takes 1.0 min to turn through 20 complete revolutions before reaching its final speed. (a) What was its angular acceleration. (b) What was its final angular speed?

One way α can be determined is this. The average angular velocity  is given by Δθ/Δt. And this is:



The average angular velocity is also given by:



equating these two expression, we can solve for the final angular velocity,



and the angular acceleration is:

